Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

6541657198

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **October/November 2023**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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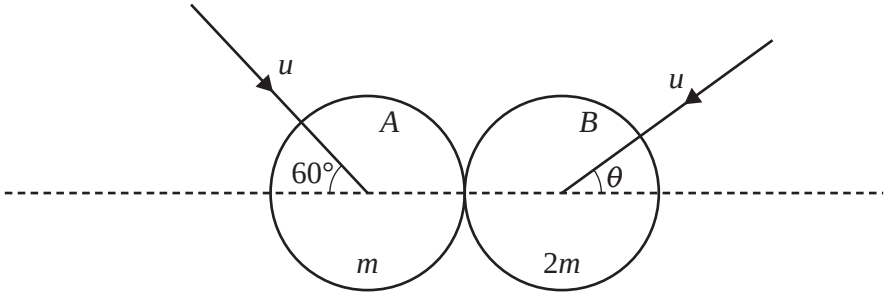

Two uniform smooth spheres A and B of equal radii have masses m and $2m$ respectively. The two spheres are moving with equal speeds u on a smooth horizontal surface when they collide. Immediately before the collision, A's direction of motion makes an angle of 60° with the line of centres, and B's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is e.

After the collision, the component of the velocity of A along the line of centres is v and B moves perpendicular to the line of centres. Sphere A now has twice as much kinetic energy as sphere B.

(a) Show that $v = \frac{1}{2}u(4\cos\theta - 1)$. [1]

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(b) Find the value of $\cos\theta$. [4]

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(c) Find the value of e. [2]

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2 A ball of mass 2 kg is projected vertically downwards with speed $5\,\text{m}\,\text{s}^{-1}$ through a liquid. At time t s after projection, the velocity of the ball is $v\,\text{m}\,\text{s}^{-1}$ and its displacement from its starting point is x m. The forces acting on the ball are its weight and a resistive force of magnitude $0.2v^2$ N.

(a) Find an expression for v in terms of t. [6]

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(b) Deduce what happens to v for large values of t. [1]

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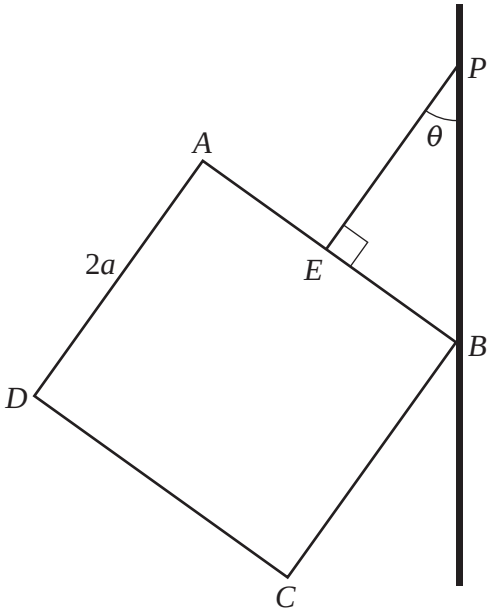

A uniform square lamina of side $2a$ and weight W is suspended from a light inextensible string attached to the midpoint E of the side AB. The other end of the string is attached to a fixed point P on a rough vertical wall. The vertex B of the lamina is in contact with the wall. The string EP is perpendicular to the side AB and makes an angle θ with the wall (see diagram). The string and the lamina are in a vertical plane perpendicular to the wall. The coefficient of friction between the wall and the lamina is $\frac{1}{2}$.

Given that the vertex B is about to slip up the wall, find the value of $\tan\theta$. [8]

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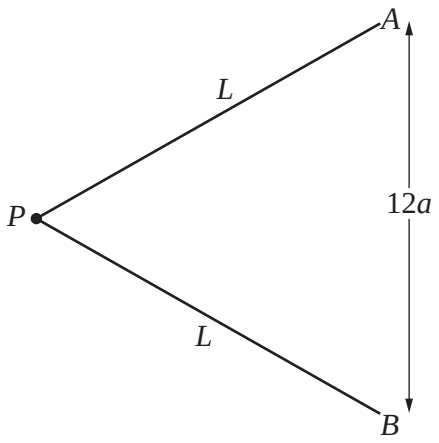

A light elastic string has natural length $8a$ and modulus of elasticity $5mg$. A particle P of mass m is attached to the midpoint of the string. The ends of the string are attached to points A and B which are a distance $12a$ apart on a smooth horizontal table. The particle P is held on the table so that $AP = BP = L$ (see diagram). The particle P is released from rest. When P is at the midpoint of AB it has speed $\sqrt{80ag}$.

(a) Find L in terms of a. [5]

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(b) Find the initial acceleration of P in terms of g. [3]

5 A particle P is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. During its flight P passes through the point which is a horizontal distance $3a$ from O and a vertical distance $\frac{3}{8}a$ above the horizontal plane. It is given that $\tan\theta = \frac{1}{3}$.

(a) Show that $u^2 = 8ag$. [2]

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A particle Q is projected with speed $V\,\mathrm{m\,s^{-1}}$ at an angle α above the horizontal from O at the instant when P is at its highest point. Particles P and Q both land at the same point on the horizontal plane at the same time.

(b) Find V in terms of a and g. [7]

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6 A particle P of mass m is attached to one end of a light inextensible rod of length $3a$. An identical particle Q is attached to the other end of the rod. The rod is smoothly pivoted at a point O on the rod, where $OQ = x$. The system, of rod and particles, rotates about O in a vertical plane.

At an instant when the rod is vertical, with P above Q, the particle P is moving horizontally with speed u. When the rod has turned through an angle of 60° from the vertical, the speed of P is $2\sqrt{ag}$, and the tensions in the two parts of the rod, OP and OQ, have equal magnitudes.

(a) Show that the speed of Q when the rod has turned through an angle of 60° from the vertical is $\frac{2x}{3a-x}\sqrt{ag}$. [2]

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(b) Find x in terms of a. [5]

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(c) Find u in terms of a and g. [4]

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Additional page

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